

Structural Mechanics

Analytical and Numerical Approaches for Structural Analysis



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In Memory of Professor Lingyi Lu

Preface

On 13th of July, 2019, I was informed about sad news of the passing away of Professor Lingyi Lu, after battling pancreatic cancer for more than half a year.

Just two days before this, I visited him at a hospital in Shanghai, and he informed me about the progress of this book and asked me to finalize it. Together with efforts of Dr. Zhuo Tang, a former PhD student of Prof. Lu, the book has finally been finished with a maximum preservation of Professor Lu's academic thoughts.

This book covers essential topics on structural mechanics and dynamics. It is mainly based on the contents of an undergraduate course in structural mechanics taught by Professor Lu at Southeast University, from the 1990s in the last century to 2019, and is an important contribution of Professor Lu's lifetime academic and pedagogic activities that he has been deeply engaged in, reflecting the forefront education status of structural mechanics in China during that period.

Modern structural mechanics originated from Galileo Galilei, Isaac Newton, and Robert Hooke. It is the computation of deformations, deflections, and internal forces or stresses within structures, either for design or for performance evaluation of existing structures. With the objective of providing principles of structural mechanics, topics covered in this book include both basic and advanced ones. Basic topics include geometric stability, internal forces and deflections of statically determinate structures, force and displacement method, and influence lines. Advanced topics include matrix displacement method for structural analysis, dynamics of structures, and limit load analysis.

The book serves as a classroom textbook in structural mechanics. It is written in such a way that it can be followed by anyone with a basic knowledge of classical and material mechanics.

While the book does not seek to promote any specific "school of thought," it inevitably reflects the authors' best practices and working habits. This is particularly apparent in the topics selected and the level of detail devoted to each of them, the choice of mathematical treatments and symbolic notations. It should not deter readers from seeking to find their own best practices and working habits.

Many individuals have provided valuable supports to finalize the book. Those contributions come from the family members of Professor Lu: his wife, Professor Hong Zhuang, and his daughter, Hongling Lu, and from his former students: Hongsheng Wang, Jie Zhang, Yuanzhi Liu, Min Chen, and Wensong Hu.

Professor Gang Wu from Southeast University has inquired about the book status several times and expressed an inspiring encouragement on behalf of Southeast University.

When I visited Professor Lu in the hospital in his last stage, he had requested me to write a preface for this book, as both his friend and former undergraduate student, I came up with this preface with emotion, and with a deep memory of my undergraduate study at Southeast University, that has had a strong influence on my life and career after I left the University. Hope this book will be useful for both students and academic professionals.

Junbo Jia

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