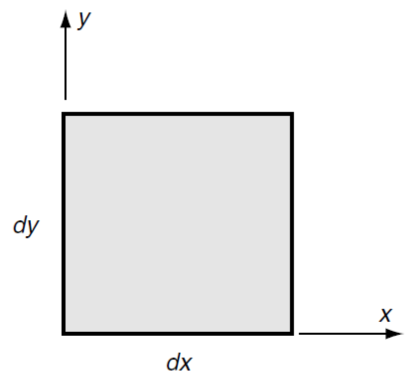
1. Determine the strain and rotation tensors for the following displacement field:

$$u = Axy, \quad v = Bxz^2, \quad w = C\left(x^2 + y^2\right)$$

where A, B, and C are constants.

2. A two-dimensional displacement field is given by $u = k\left(x^2 + y^2\right), \quad v = k\left(2x - y\right), \quad w = 0$, where k is a constant. Determine and plot the deformed shape of a differential rectangular element originally located with its left bottom corner at the origin as shown. Finally, calculate the rotation component ω_z.



3. A three-dimensional elasticity problem of a uniform bar stretched under its own weight gives the following strain field:

$$\varepsilon_{ij} = \begin{bmatrix} Az & 0 & 0 \\ 0 & Az & 0 \\ 0 & 0 & Bz \end{bmatrix}$$

where *A* and *B* are constants. Integrate the strain-displacement relations to determine the displacement components and identify all rigid-body motion terms.

4. Determine the spherical and deviatoric strain tensors for the strain field given below. Justify that the first invariant or dilatation of the deviatoric strain tensor is zero. What does the vanishing of the dilatation imply?

$$\varepsilon_{ij} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & -4 & 1 \\ 0 & 1 & 6 \end{bmatrix} \times 10^{-3}$$

5. (Optional) Using MATLAB, determine the principal values and directions of the following state of strain:

$$\varepsilon_{ij} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & -4 & 1 \\ 0 & 1 & 6 \end{bmatrix} \times 10^{-3}$$