1. For the given matrix and vector

$$a_{ij} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \qquad b_i = \begin{Bmatrix} 1 \\ 0 \\ 2 \end{Bmatrix}$$

Compute the following quantities: $a_{ii}, a_{ij}a_{ij}, a_{ij}a_{jk}, a_{ij}b_j, a_{ij}b_ib_j, b_ib_j, b_ib_i$. For each quantity, point out whether the result is a scalar, vector, or matrix. Note that $a_{ij}b_j$ is actually the matrix product $[a]\{b\}$, while $a_{ij}a_{jk}$ is the product $[a][a]$.

2. Show the following results invovling Kronecker delta (δ_{ij}) and alternating or permutation symbol (ε_{ijk}).

(a) $\delta_{ii} = 3$

(b) $\delta_{ij}\delta_{ij} = 3$

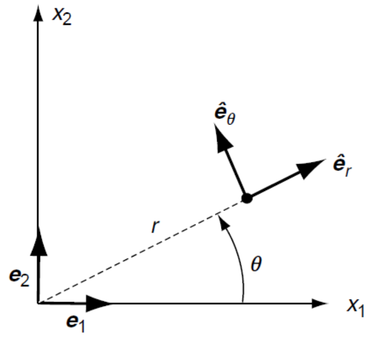
(c) $\varepsilon_{ijk}\varepsilon_{jki} = 6$

(d) $\varepsilon_{ijk}A_jA_k = 0$

(e) $\delta_{ij}\delta_{jk} = \delta_{ik}$

(f) $\delta_{ij}\varepsilon_{ijk} = 0$

3. Consider the two-dimensional coordinate transformation shown below. Through the counterclockwise rotation $\theta = 30^0$, a new polar coordinate system is created. If $b_i = \begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix}, a_{ij} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ are the components of a first- and second-order tensor in the x_1, x_2 system, calculate their components in the rotated polar coordinate system.



4. Determine the invariants, principal values, and directions of the matrix

$$a_{ij} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Use the determined principal directions to establish a principal coordinate system and formally transform (rotate) the given matrix into the principal system to arrive at the appropriate diagonal form.

5. (**Optional**) A second-order symmetric tensor field is given by

$$a_{ij} = \begin{bmatrix} 2x & x & 0 \\ x & -6x^2 & 0 \\ 0 & 0 & 5x \end{bmatrix}$$

Using MATLAB (or similar software), investigate the nature of the variation of the principal values and directions over the interval $1 \le x \le 2$. Formally plot the variation of the absolute value of each principal value over the range $1 \le x \le 2$.