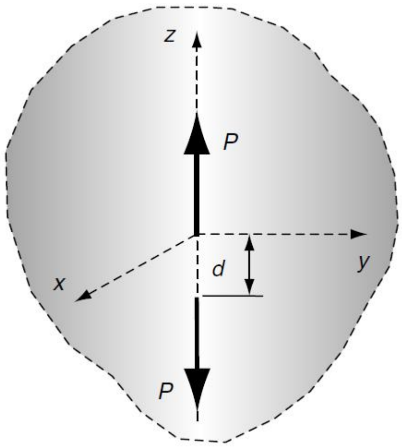
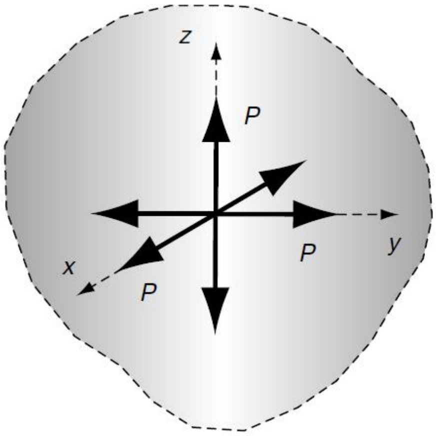
1. For the case of zero body forces, the Galerkin vector is biharmonic. However, in curvilinear coordinate systems, the individual Galerkin vector components might not necessarily be biharmonic. Consider the cylindrical coordinate case where $\mathbf{V} = V_r\mathbf{e}_r + V_\theta\mathbf{e}_\theta + V_z\mathbf{e}_z$. Using the results of tensor calculus, show that only the component V_z will satisfy the scalar biharmonic equation.

2. A *force doublet* is commonly defined as two equal but opposite forces acting in an infinite medium as shown in the figure. Develop the stress field for this problem by superimposing the Kelvin's solution onto that of another single force of $-P$ acting at the point $z = -d$. In particular, consider the case as $d \rightarrow 0$ such that the product $Pd \rightarrow D$ where D is a constant. This summation and limiting process yield a solution that is simply the derivative of the original Kelvin state. Also, using the transformation relations, develop the stress components in spherical coordinates.



3. Using the results of the previous problem, continue the superposition process by combining three force doublets in each of the coordinate directions. This results in a *center of dilatation* at the origin as shown in the figure. Using spherical coordinate components show that the stress field for this problem is symmetric with respect to the origin.

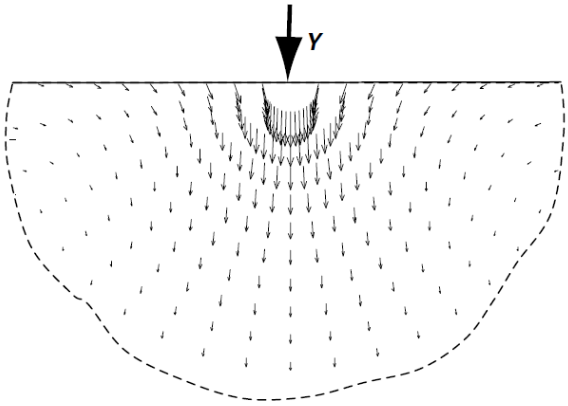


4. Explicitly show that Boussinesq's problem as illustrated in class notes is solved by the superposition of a Galerkin vector and Lamé's potential given below. Develop the Cartesian displacements and stresses.

$$V_x = V_y = 0, \quad V_z = AR; \qquad \phi = B\log(R+z).$$

5. Using the transformation relations and the results of the previous problem, develop the displacement components in cylindrical coordinates for the Boussinesq's problem. For this case, construct a displacement vector distribution plot, similar to that of the two-dimensional Flamant solution shown below. For convenience, choose the coefficient $P/4\pi G = 1$ and take $\nu = 0.3$.

   Compare the two-and three-dimensional results.



Displacement field for Flamant problem.